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Research Article

An Actuarial Comparison of Loss-Ratio-Bornhuetter-Ferguson and Classical Chain Ladder Techniques in Insurance Loss Reserving: A Computational Approach

Ogungbenle Gbenga Michael*
Department of Actuarial Science, University of Jos, Nigeria

Abstract: An insurance firm promises to reimburse benefits to the insured when unforeseen events evolve. When such contingencies arise, the underwriter has a liability to pay the claim through loss reserving techniques. The estimation of such loss reserving should be technically performed so the insurance firm will not run into a loss. This study aims to (i) estimate the Bornheutter-Ferguson reserve and the inverse of its development factor λ_n under the aegis of the loss ratio framework. (ii) numerically estimate the chain ladder reserve and final losses, (iii) estimate the Ratio of cumulative payments in successive development periods, and (iv) demonstrate to professional insurance firms how to use these techniques in practice. These techniques evaluated through some run-off loss matrix can be employed to estimate technical provisions for the outstanding claims. Computational evidence from our results over the periods analyzed confirms that on the assumption of 0.9 the loss ratio, the Bornheutter-Ferguson technique as an actuarial extension of the chain ladder method numerically presents a lower reserve value than the corresponding chain ladder reserve, and hence $CL_{reserve}$=249,811.708>$BF_{reserve}$=86,612.58.

Keywords: Chain ladder, Bornheutter-Ferguson, Loss-ratio, Run-off triangles.

1. Introduction

A fundamental problem in the general insurance business underwriting business is concerned with the computation of technical provisions with respect to the unpaid liabilities of an underwriter to the insured. Hence, we intend to demonstrate to the insurance firms and the academic community useful, practical steps of actuarial reserving, which are difficult for many to understand. Wiser (1990) developed the mathematical techniques for estimating these liabilities comprising a varying range of categories that have been modeled to mirror exact numerical algorithms whose essence is to satisfy target requirements. Suwardi and Purwono (2020) complicated estimation of reasonable structural loss reserving that is based on run-off triangles and the appraisal of full information on the actual claim procedure seems to be an intractable challenging area of core actuarial non-life mathematical provisions. A sound basis of the variation of underwriting principles and data processing procedures affecting the underwriter's experience is indispensable to the precise estimation and appraisal of observed data in the presence of the chosen reserving techniques (Adam, 2018). Let $g : \mathbf{R}^+ \to \mathbf{R}^+; \xi \mapsto g(\xi)$ such that $g(\xi)$ defines a continuous claim density function representing the value of a claim momentarily at time ξ and suppose further $g(\xi)$ is a function which specifies the rate at which claim occurs instantaneously at time ξ. We assume $g(\xi)$ is expected to demonstrate how claims occur as obtained by the in-force risk exposure of the distribution of the risk exposure. The mathematical expectation of $g(\xi)$ is usually a function of the evolving premium pattern, and as such, the expected value would be computed as an in-force exposure basis. From the foregoing, the aggregate claim $G(s,\tau)$which occurs within the time interval $s < \xi < \tau$ is obtained as $G(s,\tau) = \int_s^\tau g(\xi)\, d\xi$, and this integral is the ultimate loss incurred in the interval $s < \xi < \tau$.The loss development function $\theta(\tau)$ is a function that specifies the fraction of losses that have been paid within τ years after the loss has occurred. It is then apparent that $\lim\limits_{\tau<0} \theta(\tau) = 0$ while $\lim\limits_{\tau\to 0} \theta(\tau) = 1$. For any specified time $t > \xi$,the function $P(t,\xi) = g(\xi) \times \theta(t-\xi)$ defines the aggregate paid amount t , for losses occurring at time ξ.

We assume that the function $P(t,\xi)$ is continuous, so the aggregate paid value could be integrated over the interval $s < \xi < \tau$.Therefore, given the loss function $g(\xi)$ and the loss development function $\theta(\xi)$,paid losses from the losses incurred in interval $s < \xi < \tau$ as developed to time $s \le \tau < t$ will be obtained as

$$G(t,s,\tau) \int_s^\tau g(\xi) \times \theta(t-\xi)\, d\xi \tag{1}$$

and this describes the paid claims over the interval $s \le \tau < \tau$ and, consequently, the function $U(t,s,\tau)$ defined by

$$\begin{aligned} U(t,s,\tau) &= G(s,\tau) - G(t,s,\tau) \\ &= \int_s^\tau g(\xi)\, d\xi - \int_s^\tau g(\xi) \times \theta(t-\xi)\, d\xi \end{aligned} \tag{2}$$

specifies the unpaid losses. Again, given the loss function $g(\xi)$ and the loss development function $\theta(\xi)$ then the loss reserves for claims incurred in the interval $s < \xi < \tau$ as developed to time $s \le \tau < t$ will be obtained as

**Corresponding Author's Email: moyosiolorun@gmail.com*

$$
\begin{aligned}
r(t) &= \int_s^\tau g(\xi)\, d\xi - \int_s^\tau g(\xi) \times \theta(t-\xi)\, d\xi \\
&= \int_s^\tau g(\xi) \{1 - \theta(t-\xi)\}\, d\xi
\end{aligned}
\tag{3}
$$

But where the reserves are to be discounted through the present value, it is sufficient to embed the affine interest rate intensity model $\delta(t)$ at time t as the discounted factor defined by

$$
d\delta(t) = \alpha(\beta - \delta(t))\, dt + \sigma dW(t) \tag{4}
$$

where $t \to W(t)$ is the standard one-dimensional Brownian motion and α the speed of reversion, β-the long-term mean level, and σ-the volatility of interest rate while the drift $\alpha(\beta - \delta(t))$ is the instantaneous change in interest rate at time ξ.The equation (4) is solved

$$
\begin{aligned}
\delta(t) =& \delta(0) e^{-at} + \int_0^t \alpha\beta e^{-\alpha(t-U)} dU \\
&+ \sigma \int_0^t e^{-\alpha(t-U)} dW(U)
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
\delta(\tau + t) =& \delta(0) e^{-\alpha(\tau+t)} + \int_0^{\tau+t} \alpha\beta e^{-\alpha(\tau+t-U)} dU \\
&+ \sigma \int_0^{\tau+t} e^{-\alpha(\tau+t-U)} dW(U).
\end{aligned}
\tag{6}
$$

The aggregate interest rate intensity in the interval $0 \leq \xi < t + \tau$ is obtained as $\bar{\delta}(t+\tau) = \int_0^{t+\tau} \delta(\xi)\, d\xi$ and setting $t = 0$, the aggregate interest rate intensity in the interval $0 \leq \xi < \tau$ becomes $\bar{\delta}(\tau) = \int_0^{\tau} \delta(\xi)\, d\xi$ so that the change of interest rate in the time interval is given as

$$
\begin{aligned}
\bar{\delta}(t+\tau) - \bar{\delta}(t) &= \int_0^{t+\tau} \delta(\xi)\, d\xi - \int_0^{\tau} \delta(\xi)\, d\xi \\
&= \int_t^{t+\tau} \delta(\xi)\, d\xi
\end{aligned}
\tag{7}
$$

Let $\dfrac{\mathrm{d}(\xi)\theta}{\mathrm{d}\xi} = \phi(\xi)$ implies that

$$
\frac{\mathrm{d}(V-\xi)\theta}{\mathrm{d}(V-\xi)} = \phi(V-\xi) \tag{8}
$$

and suppose ξ is a fixed time for $s \leq \xi < \tau$ such that a loss $g(\xi)$ is incurred. Then $g(\xi) \times \phi(V-\xi)\, dV$ of the losses will evolve momentarily for $V > \xi$. Consequently $g(\xi) \times \phi(V-\xi)\, dV$ would imply that $g(\xi) \times \dfrac{\mathrm{d}(\theta(V-\xi)}{\mathrm{d}(V-\xi)}$. Therefore, using the interest rate intensity, the present value at a specified time t within the interval $\xi \leq t \leq V$ is obtained as

$$
\begin{aligned}
&\{g(\xi) \times \phi(V-\xi)\, dV\} \times e^{-\left\{\left(\int_0^V \delta(U)dU - \int_0^\xi \delta(U)dU\right)\right\}} \\
&= \{g(\xi) \times \phi(V-\xi)\, dV\} \times e^{-\int_0^V \delta(U)dU + \int_0^\xi \delta(U)dU} \\
&= \{g(\xi) \times \phi(V-\xi)\, dV\} \times e^{\int_V^\xi \delta(U)dU}
\end{aligned}
\tag{9}
$$

We can now integrate overall future development times $t \leq V \leq \infty$ to determine the discounted value of the unpaid loss reserves

$$
\Phi(t) = \int_s^\tau g(\xi) \int_t^\infty \left(e^{\int_V^\tau} \delta(U)\, dU\right) \frac{\partial\theta(V-\xi)}{\partial\xi} dV d\xi \tag{10}
$$

at a time t from the interval (s, τ) as developed to

$$
s \leq \tau < t. \tag{11}
$$

Observe that $\lim_{\xi\to\infty} \theta(V-\xi) = 1$ by definition. Therefore, from the $\Phi(t)$ integral, if we set $\Delta(U) = 0$ then, we have

$$
\begin{aligned}
\Phi(t) &= \int_s^\tau g(\xi) \int_t^\infty \frac{\partial\theta(V-\xi)}{\partial\xi} d\xi dV \\
&= \int_s^\tau g(\xi) \left(\lim_{\xi\to\infty} \theta(V-\xi) - \theta(V-t)\right) dV \\
&= \int_s^\tau g(\xi)(1 - \theta(V-t))\, dV
\end{aligned}
\tag{12}
$$

which is a form of the reserve

$$
r(t) = \int_s^\tau g(\xi) \{1 - \theta(t-\xi)\}\, d\xi \tag{13}
$$

2. Methodology

2.1. Chain Ladder Estimation

The chain ladder technique is an algorithm-based claim reserving technique used to obtain loss reserves. General professional insurance firms could employ the information used through this technique to assist their enterprise risk functions by computing different risk measures. Insurance firms usually estimate the requisite surplus to

Table 1: The actuarial basis of run-off triangle

	Develop year						
Accident year	1	2	...	j	...	J-1	J
1	$M_{1,1}$	$M_{1,2}$	...	$M_{1,j}$	...	$M_{1,J-1}$	$M_{1,J}$
2	$M_{2,1}$	$M_{2,2}$	...	$M_{2,j}$	...	$M_{2,J-1}$	$M_{2,J}$
...							
i	$M_{i,1}$	$M_{i,2}$	...	$M_{i,j}$	..	$M_{i,J-1}$	$M_{i,J}$
...					$i+j \geq I$		
$I-1$	$M_{I-1,1}$	$M_{I-1,2}$	.	$M_{I-1,j}$		$M_{I-1,j-1}$	$M_{I-1,J}$
I	$M_{I,1}$	$M_{I,2}$		$M_{I,j}$		$M_{I,J-1}$	$M_{I,J}$

distribute to their stakeholders towards the end of the financial period. The underwriter must determine the size of a buffer fund necessary to pay all outstanding claims over which full premiums have been received to enable them to estimate the required surplus (Weindorfer, 2012; Adam,2018; Carvalho and Carvalho, 2019; Suwardi and Purwono, 2020). An appreciable time may elapse between the time a claim occurs and the time its agreeable value is fully settled because a closed claim can be revisited for litigation, or the process of advising a claim to the insurance may have been delayed. Usually, an underwriter investigates all claims advised through its loss adjusters to determine their genuineness, which may prolong settlement. In Table 1, the claim advised is presented in a run-off triangular form such that the height of the triangle describes the accident year i of the claim, and the base of the triangle is the development year j of the claim. I is the previous year that the claim occurred, while J describes the highest number of development years. In many cases, $J = I$. Now, assuming that for the accidental year i and development year j, the incremental claim value is defined by $M_{i,j}$. The upper triangle of the run-off can be defined as

$$T_I^U = \{M_{ij} : i + j \leq I; 0 \leq j \leq J\} \tag{14}$$

while the lower triangle is defined as

$$T_I^U = \{M_{ij} : i + j \geq I; 0 \leq j \leq J\} \tag{15}$$

Each M_{ij} describing the incremental claim in the run-off triangle is obtained as

$$M_{ij} = S_i \times \theta_j \times y_{i+j} + \mu_{ij} \tag{16}$$

.

In the description above, θ_j is the development factor for the year j and defines the percentage of claim payments in development year j. The θ_j is independent of the accident year i of the claim. The S_i is a parameter changing by accident year i and defines the exposure for the number of claims incurred in accident year i. The term y_{i+j} measures inflation indices for the calendar year, while μ_{ij} is the noise term.

The $M_{i,j}$ is either the incremental claim number or the incremental claim amount. We can then present either the cumulative incremental claim number or the cumulative claim amount by $T_{i,j} = \sum_{n=1}^{j} M_{i,n}$ where i is the accident year and j is the development year. The $M_{i,j}$ is only observed when $i + j < I$, which are the observed claims, and then defines the upper run-off triangle. The lower run-off triangle defines the claims value $M_{i,j}$ for $i + j > I$ that is to be estimated. Therefore, the outstanding claim reserve r_i for accident year I can be obtained as $r_i = T_{i,j} - T_{i,n-i+1}$ for $1 \leq j \leq J$. Consequently, the total outstanding claim reserve becomes $r = \sum_{i=1}^{I} r_i$, and this is what the underwriting firm must reserve. The chain ladder algorithm assumes that each accident year follows a similar trend of claim development. It also assumes a weighted average of past values when inflation in the previous years is certain to be recurrent in the future years because inflation could be a key fiscal variable that is being carried forward into the future through development factors. The development factors are obtained as

$$\hat{G}_j = \frac{\sum_{i=1}^{i-j+1} M_{i,j}}{\sum_{i=1}^{i-j+1} M_{i,j-1}} \tag{17}$$

The development factors are hence applied in computing the future cumulative claim reserves by applying them to the cumulative claims on each row. $\widehat{M_{i,j-i+2}} = M_{i,j-i+1} \times \hat{G}_{j-i+2}$ for $2 \leq j \leq J$ and for the nth row, we obtain

$$\hat{M}_{i,n} = \hat{M}_{i,n-i} \times \hat{G}_j \tag{18}$$

for $2 \leq j \leq J$ and for $3 - i + m \leq n \leq m$.

3. Data Analysis

The data used in our computation was sourced from the historical loss development study covering the periods 2010 to 2019 and published by MathWorks (2021).

3.1. Credibility Premium

Credibility theory is an actuarial framework under which issues bordering on how much we depend on the claims

Table 2: Past claims for five different policies

	Policy j				
Year i	1	2	3	4	5
2016	0	5	2	5	3
2017	0	1	1	3	0
2018	0	1	0	2	0
2019	0	1	1	2	1
Total	0	8	4	12	4
Mean	0	2	1	3	1
Variance	0	4	0.6667	2	2

experiences of other homogeneous lines of business and how much weight that is expected to be assigned to the observations of such individual lines of business being analyzed are addressed. The three key variables stated in the premium equation are (i) credibility factor α, (ii) individual premium $\bar{x_j}$, and (iii) collective premium ϕ needed to compute credibility premium for five different auto insurance policies.

$$(1-\alpha)\,(\textbf{Collective premium}) = \textbf{Collective premium} - \alpha\,(\textbf{Individual premium}) \tag{19}$$

where α is the credibility factor and $0 \leq \alpha \leq 1$.We used the previous information about the volume of accidents in 4 past years. For each policy, we first compute individual premiums, which are equivalent to the average claim for four years, using

$$\bar{x_j} = \frac{1}{4}\sum_{i=1}^{4} x_{ij} \tag{20}$$

where $\bar{x}$ is the individual premium i in the year and j is the policy.

The calculated credibility premium for five different auto insurance contracts. To do so, we need three numbers: (i) credibility factor $\left(\hat{\beta}\right)$, (ii) Individual premium $\bar{x_j}$ and (iii) Collective premium ψ.We have past information about the number of accidents for four previous years. For each policy, we first calculate the individual premium, which equals the average claim for four years, using equation (20). Where: $\bar{x}$ = individual premium, i = year, and j = policy.

Next, we calculate collective premium using equation (21).

$$\gamma = \frac{1}{k \times l} \times \sum_{j=1}^{l}\sum_{i=1}^{k} x_{ij} = \frac{1}{l}\sum_{j=1}^{l}\left(\frac{1}{k}\sum_{i=1}^{k} x_{ij}\right) \tag{21}$$

$$\gamma = \frac{1}{5 \times 4}\,(0+8+4+12+4) = \frac{28}{20} = 1.4 \tag{22}$$

where: $k=$ number of years, $l=$ number of contracts, and $\gamma=$ collective premium We have individual premiums and collective premiums; we only need to get the credibility factor, denoted by $\hat{\beta}$, to calculate credibility premiums. The credibility factor is:

$$\hat{\beta} = \frac{k\psi}{k\psi + \hat{W}} \tag{23}$$

Following Straub (1988), $\hat{\beta}$ = credibility factor, $\hat{W}$ = variance from year to year, and ψ variance from risk policy to risk policy. In order to determine the credibility factor, it is necessary to ascertain the values of the variances $\hat{W}$ and ψ. As observed from the equation for credibility factors and credibility premium, we can confirm that the higher the variance of the individual premium, the greater the denominator of the credibility factor, which implies a smaller credibility factor, and hence, lower weight is mapped to the individual premium. Consequently, the parameter $\hat{W}$ measures the variance of each scheme. However, the bigger the variance between the policies, the smaller the denominator of the credibility factor becomes and the higher the credibility factor; hence, a smaller weight is assigned to the collective premium. Consequently, ψ measures heterogeneity among the schemes. We compute $\hat{W}$ to ensure our computation becomes convenient. Observe that we have computed the variance of each premium in Table 2; hence, we only need to compute the average of the variances to get $\hat{W}$.

$$\hat{W} = \frac{1}{L}\sum_{j=1}^{L}\frac{1}{k-1}\sum_{i=1}^{K}\left(x_{ij} - \bar{x_j}\right)^2 \tag{24}$$

$$\hat{W} = \frac{1}{5}\,(0+4+0.6667+2+2) \tag{25}$$

Table 3: Credibility premium

Policy j	$\hat{x}_j$	$\hat{u}_j$
Policy 1	0	0.6667×0+(1-0.6667)×1.4=0.4666
Policy 2	2	0.6667×2+(1-0.6667)×1.4=2.3334
Policy 3	1	0.6667×1+(1-0.6667)×1.4=1.1333
Policy 4	3	0.6667×3+(1-0.6667)×1.4=2.4667
Policy 5	1	0.6667×1+(1-0.6667)×1.4=1.1333

Table 4: Cumulative run-off Triangle

		Development years j				
		0	1	2	3	... f
Accident years i	2010	$S_{2010,0}$	$S_{2010,1}$	$S_{2010,2}$	$S_{2010,3}$	$S_{2010,f}$
	2017	$S_{2017,0}$	$S_{2017,1}$	$S_{2017,2}$	$S_{2017,3}$	
	2018	$S_{2018,0}$	$S_{2018,1}$	$S_{2018,2}$		
	2019	$S_{2019,0}$	$S_{2019,1}$			
	f	$S_{f,0}$				

$$\hat{W} = \frac{1}{5} \times 8.6667 = 1.7333 \tag{26}$$

Since $\hat{W}$ this has been determined, we can compute ψ as follows:

$$\psi = \frac{1}{L-1} \sum_{j=1}^{L} (\bar{x_j} - \pi)^2 - \frac{\hat{W}}{K} \tag{27}$$

$$\begin{aligned}\psi =& \frac{1}{5-1} \times [(0-1.4)^2 + (2-1.4)^2 \\ &+ (1-1.4)^2 + (3-1.4)^2 \\ &+ (1-1.4)^2] - \frac{1.7333}{4}\end{aligned} \tag{28}$$

$$\begin{aligned}\psi &= \frac{1}{4}(1.96 + 0.36 + 0.16 + 2.56 + 0.16) - 0.4333 \\ &= 1.3 - 0.4333 = 0.8667\end{aligned} \tag{29}$$

$$\psi = 0.8667 \tag{30}$$

The credibility factor then becomes

$$\hat{\beta} = \frac{4}{4 + \dfrac{1.7333}{0.8667}} = \frac{4}{5.9999} = 0.6667 \tag{31}$$

The credibility factor has a value of 0.6667, implying that a more significant weight is assigned to individual experience than total experience. The reason is that the variance of each scheme is lower in comparison to the variance among schemes. Following Straub (1988), the information garnered across each premium, collective premium, and credibility factor could be applied to compute the credibility premium for each scheme, as shown in Table 3.

$$\begin{aligned}\hat{u_j} &= \hat{\beta} + \hat{x}_j + \left(1 - \hat{\beta}\right) \times \gamma \\ &= 0.6667 + (1 - 0.6667) \times 1.4 = 1.13332\end{aligned} \tag{32}$$

4. Chain-Ladder Method

The Chain Ladder method is the first deterministic technique developed to compute technical provisions for incurred but unreported claims (IBNR). In this method, the actuary predicts the future expected claims from earlier paid or reported claims. The chain ladder assumes that the time series of claims is stable in time. For the data input, we need a run-off triangle that collects cumulative data over the incurred claims with respect to the accident year and development year. The accident year is the year in which the accident occurred, and the development year defines how claims are paid in years progressively after the accident. Usually, a delay could occur between claim occurrence and claim settlement. Cumulative claims are determined by S_{ij}, where i = accident year and j = development year x_{ji} then the total claim is $X_{2010,0} + X_{2010,1} + X_{2010,2} + X_{2010,3} + \cdots + X_{2010,f}$.

In Table 4, we already have cumulative claims, but a run-off loss triangle with incremental claims could be applied. The unknown part of the loss triangle is estimated by applying the development factors. The development

Table 5: Incremental run-off Triangle

	0	1	2	3	4	5	6	7	8	9
2010	3995.71	4635	4866.78	4964.1	5013.74	5038.82	5058.97	5074.14	5084.29	5089.38
2011	3968.04	4682.3	4963.22	5062.49	5113.11	5138.67	5154.09	5169.56	5179.89	
2012	4217.01	5060.4	5364.04	5508.87	5556.45	5586.24	5608.59	5625.41		
2013	4374.24	5205.3	5517.67	5661.12	5740.38	5780.56	5803.68			
2014	4499.68	5309.6	5628.2	5785.79	5849.43	5878.68				
2015	4530.24	5300.4	5565.4	5715.66	5772.82					
2016	4572.63	5304.3	5569.47	5714.27						
2017	4680.56	5523.1	5854.44							
2018	4696.68	5495.1								
2019	4945.89									

Table 6: Cumulative run-off Triangle

	0	1	2	3	4	5	6	7	8	9
2010	3995.71	8630.73	13497.5	18461.61	23475.35	28514.17	33573.14	38647.28	43731.57	48820.95
2011	3968.04	8650.32	13613.5	18676.03	23789.14	28927.81	34081.9	39251.46	44431.35	
2012	4217.01	9277.43	14641.5	20150.34	25708.79	31295.03	36903.62	42529.03		
2013	4374.24	9579.58	15097.3	20758.37	26498.75	32279.31	38082.99			
2014	4499.68	9809.3	15437.5	21223.29	27072.72	32951.4				
2015	4530.24	9830.62	15396	21111.68	26884.5					
2016	4572.63	9876.88	15446.4	21160.62						
2017	4680.56	10203.6	16058.1							
2018	4696.68	10191.8								
2019	4945.89									

Table 7: Ratio of cumulative payments in successive development periods

	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
2010	2.1600	1.5639	1.3678	1.2716	1.2146	1.1774	1.1511	1.1316	1.1164
2011	2.1800	1.5738	1.3719	1.2738	1.2160	1.1782	1.1517	1.1320	
2012	2.2000	1.5782	1.3762	1.2758	1.2173	1.1792	1.1524		
2013	2.1900	1.5760	1.3750	1.2765	1.2181	1.1798			
2014	2.1800	1.5738	1.3748	1.2756	1.2171				
2015	2.1700	1.5661	1.3712	1.2734					
2016	2.1600	1.5639	1.3699						
2017	2.1800	1.5738							
2018	2.1700								
2019									

factor is given in equation (33) as follows

$$\hat{\lambda_j} = \frac{\sum_{i=0}^{f-1-j} S_{ij+1}}{\sum_{i=0}^{f-1-j} S_{ij}} \tag{33}$$

Following Olivieri and Pitacco (2010), the development factor shows for any accident year i the increase of the cumulative aggregate claim from time j to time $j+1$, assuming the claims are fully covered till the year f then $\hat{\lambda_f}$. A final development factor is computed for every development year by multiplying the estimated development factors. The final development factor is expressed as:

$$\hat{H_j} = \lambda_j \times \lambda_{j+1} \times \lambda_{j+2} \times \ldots \times \lambda_{f-1} \tag{34}$$

The final losses are computed using equation (35).

$$S_{if} = S_{ij} \times \hat{H_j} \tag{35}$$

The input data for our computation is an incremental run of the triangle in Table 5, which contains data about claims from year 2010 to 2019.

For the claims originating in 2010, payments totaling 3,995.71 were made that same year (development year 0), while payments totaling 4,635.00 were disbursed in the following year, 2011 (development year 1). The cumulative paid claims are given in Table 6. The top row is the development year j, while the first column is the accident year i. The claims in the incremental run-off triangle above are summed to calculate the cumulative run-off triangle.

Again, the top row is the development year j, while the first column is the accident year i The claim payments must be estimated after 2019 for the years of origin 2011, 2012, 20132014, 2015, 2016, 2017, 2018, and 2019 to

Table 8: Ratio of cumulative payments in successive development periods completed

	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
2010	2.1600	1.5639	1.3678	1.2716	1.2146	1.1774	1.1511	1.1316	1.1164
2011	2.1800	1.5738	1.3719	1.2738	1.2160	1.1782	1.1517	1.1320	**1.1200**
2012	2.2000	1.5782	1.3762	1.2758	1.2173	1.1792	1.1524	**1.1300**	**1.1200**
2013	2.1900	1.5760	1.3750	1.2765	1.2181	1.1798	**1.1500**	**1.1300**	**1.1200**
2014	2.1800	1.5738	1.3748	1.2756	1.2171	**1.1800**	**1.1500**	**1.1300**	**1.1200**
2015	2.1700	1.5661	1.3712	1.2734	**1.2200**	**1.1800**	**1.1500**	**1.1300**	**1.1200**
2016	2.1600	1.5639	1.3699	**1.2700**	**1.2200**	**1.1800**	**1.1500**	**1.1300**	**1.1200**
2017	2.1800	1.5738	**1.3700**	**1.2700**	**1.2200**	**1.1800**	**1.1500**	**1.1300**	**1.1200**
2018	2.1700	**1.5700**	**1.3700**	**1.2700**	**1.2200**	**1.1800**	**1.1500**	**1.1300**	**1.1200**
2019	**2.1800**	**1.5700**	**1.3700**	**1.2700**	**1.2200**	**1.1800**	**1.1500**	**1.1300**	**1.1200**

Table 9: Final development Factors

Development year j	$\hat{\lambda}_j$	$\hat{H}_j$
0	2.18	12.48
1	1.57	5.74
2	1.37	3.65
3	1.27	2.66
4	1.22	2.09
5	1.18	1.72
6	1.15	1.46
7	1.13	1.26
8	1.12	1.12

enable us to estimate the outstanding claim provisions demanded in 2019 for the years of origin. Estimating the ratios between progressive, and cumulative payments within the year of origin in Table 7 becomes necessary. This is necessary to determine the proportionate relationship between periods.

The Ratio of cumulative payments is computed from the cumulative run-off triangles using Table 6. In row 2010, we have $2.160 = \frac{8630.73}{3995.71}; 1.5639 = \frac{13497.5}{8630.73}$; and we continue in this manner till all ratio values are completely computed.

The development factors should be computed to complete the development triangle of year-to-year development ratios in Table 7. The development factors seem more stable for the cumulative payments; consequently, the development factors will be computed over cumulative payments rather than across the original yearly payments.

The development factors $\hat{\lambda}_r; \quad r = 0, 1, 2, 3, \ldots 8$ are calculated using equation (36) to forecast the unknown part of the triangle.

$$\hat{\lambda}_r = \frac{\sum_{i=0}^{9} S_{i(r+1)}}{\sum_{i=0}^{9} S_{ir}} \tag{36}$$

According to that, value of $\hat{\lambda}_0, \ \hat{\lambda}_1, \ \hat{\lambda}_2, \ \hat{\lambda}_3, \ \hat{\lambda}_4, \ \hat{\lambda}_5, \ \hat{\lambda}_6, \ \hat{\lambda}_7, \ \hat{\lambda}_8$ are 2.18, 1.57, 1.37, 1.27, 1.22, 1.18, 1.15, 1.13 and 1.12, respectively.

Table 8 shows the completed cumulative period loss triangle. We then compute the final development factors $\hat{H}_j$ using equation (37), and values are displayed in Table 9.

$$\hat{H}_j = \Pi_{r=j}^{8} \lambda_r \tag{37}$$

We then compute final losses S_{i9} and reserves R_i for the years 2010 to 2019. The final reserve represents the difference between the final loss and the last known claim. To determine the total chain ladder reserve, we sum the final reserves.

Final losses

$$\begin{aligned}
S_{16} &= S_{15} \times \hat{H}_8 = 44431.35 \times 1.12 = 49763.112 \\
S_{26} &= S_{24} \times \hat{H}_7 = 42529.03 \times 1.27 = 54011.8681 \\
S_{36} &= S_{33} \times \hat{H}_6 = 38082.99 \times 1.46 = 55601.1654 \\
S_{46} &= S_{42} \times \hat{H}_5 = 32951.40 \times 1.72 = 56676.408 \\
S_{56} &= S_{51} \times \hat{H}_4 = 26884.50 \times 2.10 = 56457.45 \\
S_{66} &= S_{60} \times \hat{H}_3 = 21160.62 \times 2.66 = 56287.2492 \\
S_{77} &= S_{70} \times \hat{H}_2 = 16058.06 \times 3.65 = 58611.919 \\
S_{88} &= S_{80} \times \hat{H}_1 = 10191.79 \times 3.72 = 37913.4588 \\
S_{99} &= S_{90} \times \hat{H}_0 = 4945.89 \times 12.48 = 61724.7072
\end{aligned} \tag{38}$$

Table 10: Calculation of development year factor λ_n

Development period	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
Development factors	2.1800	1.5700	1.3700	1.2700	1.2200	1.1800	1.1500	1.1300	1.1200
Cumulative factors	12.4772	5.7235	3.6455	2.6610	2.0953	1.7174	1.4554	1.2656	1.1200
Inverse λ_n	0.0801	0.1747	0.2743	0.3758	0.4773	0.5823	0.6871	0.7901	0.8929

Table 11: Estimated reserve

Year of origin	Premium written	Estimated ultimate claim	R_n	$1-R_n$	Estimated reserve
2011	10000	9000	0.8929	0.1071	963.9
2012	12000	10800	0.7901	0.2099	2266.92
2013	14000	12600	0.6871	0.3129	3942.54
2014	16000	14400	0.5823	0.4177	6014.88
2015	18000	16200	0.4773	0.5227	8467.74
2016	20000	18000	0.3758	0.6242	11235.6
2017	22000	19800	0.2743	0.7257	14368.86
2018	24000	21600	0.1747	0.8253	17826.48
2019	26000	23400	0.0801	0.9199	21525.66

Reserves

$$\begin{aligned}
R_1 &= S_{16} - S_{15} = 49763.112 - 44431.35 = 5331.762 \\
R_2 &= S_{26} - S_{24} = 54011.8681 - 42529.03 = 11482.8381 \\
R_3 &= S_{36} - S_{33} = 55601.1654 - 38082.99 = 17518.1754 \\
R_4 &= S_{46} - S_{42} = 56676.408 - 32951.40 = 23725.008 \\
R_5 &= S_{56} - S_{51} = 56457.45 - 26884.50 = 29572.95 \\
R_6 &= S_{66} - S_{60} = 56287.2492 - 21160.62 = 35126.6292 \\
R_7 &= S_{77} - S_{70} = 58611.919 - 16058.06 = 42553.859 \\
R_8 &= S_{88} - S_{80} = 37913.4588 - 10191.79 = 27721.6688 \\
R_9 &= S_{99} - S_{90} = 61724.7072 - 4945.89 = 56778.8172
\end{aligned} \tag{39}$$

Total chain ladder reserve is

$$\sum_{i=1}^{9} R_i = 249,811.708, \tag{40}$$

which equals the value of total expected future claims for accidents that happened between 2010 and 2019.

5. The Loss-Ratio-Bornheutter-Ferguson Technique

Following Schmidt and Zocher (2016), the Bornheutter-Ferguson technique is based on the assumption that there are parameters $\theta_1, \theta_2, \theta_3, \ldots \theta_m$ and $\xi_0, \xi_1, \xi_2, \xi_3, \ldots \xi_m$ satisfying the condition $1 = \xi_m$ and $E\left(M_{I,J}\right) = \theta_I \xi_J$ for $\{I, J\} \in \{0, 1, 2, 3, \ldots, m\}$. Therefore, $E\left(M_{I,m}\right) = \theta_I$ Consequently, $\frac{E(M_{I,J})}{E(M_{I,m})} = \xi_J$.The parameters $\xi_0, \xi_1, \xi_2, \xi_3, \ldots \xi_m$ form a development pattern for cumulative quotas. In this method, a loss ratio of a group of business lines is obtained as the Ratio of the ultimate losses to the particular premiums. During an accident year, premiums are gained, and if an underwriting year is used, the required premium equates to the premium written over the period. Reserves are determined as the product of premiums and the expected loss ratio to estimate the ultimate claim value. Although the loss ratio could be estimated after considering previous market statistics, it is widely applied to premiums. A reasonable method may be to map expected losses to a more precise measure of exposure. Since new insurance firms have little actual loss data to perform actuarial loss analysis, they should set aside reserves for outstanding claims based on loss ratios. Consequently, where the expected assumptions during which the business proposal was prepared have not significantly changed, the estimated loss ratios may be used in the firm's underwriting operations.

The first row of Table 10 is extracted from the last row of Table 8.

6. Discussion

From the result in Table 2, having predicted the individual premium and collective premium, it is necessary to evaluate the credibility factor $\hat{\beta}$.To arrive at the credibility factor, it also becomes necessary to investigate the behavior of the variances $\hat{W}$ and γ. From the equations for credibility factors and credibility premium, the bigger the variance of the individual premium, the higher the denominator of the credibility factor, which implies that the lower the credibility factor and, consequently, a smaller weight is assigned to the individual premium. Therefore, $\hat{W}$ measures the noise of individual policy. On the other hand, the higher the variance between the policies, the lower the denominator of the credibility factor, and the higher the credibility factor; therefore, the lower weight is assigned to the collective premium. Thus, γ measures the heterogeneity between policies. The credibility factor is 0.666, meaning a higher weight is given to individual experience than overall experience. This is because the variance of the individual claim is lower compared to the variance between policies. In Table 3, it becomes apparent that higher premiums have been imposed on policyholders $2, 3, 4$ and 5 compared to their average claims, while policyholders will pay smaller premiums than their average claim. If a different insurer can obtain the difference between any two sets of aggressive and defensive scheme holders, he could make different kinds of categorizations. Through this categorization, insurers could apply data analytics to know the customer behavior and better personalize insurance products and again classify aggressive and defensive scheme holders into different groups, which could advise insurers on charging high premiums to aggressive scheme holders and lower premiums to defensive shareholders. Table 6 represents the run-off triangle, which contains cumulative data about the incurred claims with respect to accident year and development year. Table 5 represents the source data, while Table 6 is the cumulative run-off triangle of the source data to provide the professional insurer a feel of the historical developments as an insight into the future. The claim payments in each of the years $2010-2019$ of the data in Table 6 were compared with the payments computed by the chain ladder method. From the Table 9, it is clear that the initial development factors are

$$D_{in} = \{2.18, 1.57, 1.37, 1.27, 1.22, 1.18, 1.15, 1.13, 1.12\} \tag{41}$$

while the final development factors are defined in equation (40) as follows

$$D_{fin} = \{12.48, 5.72, 3.65, 2.66, 2.10, 1.72, 1.46, 1.27, 1.12\} \tag{42}$$

These development factors are adopted to forecast the unknown parts of the run-off triangle. Note that the initial development factor is used to compute the final development factor. Equation (38) defines the numerical estimate of the final losses, while equation (39) defines the estimates of the chain ladder reserve. From equation (40), the chain-ladder reserve value represents the total value of the expected future claims for those accidents that evolved between the period of []. The cumulative factors of Table 10 are computed as follows.

$$\begin{aligned}
12.4772 =& 2.1800 \times 1.5700 \times 1.3700 \times 1.2700 \times 1.2200 \\
& \times 1.1800 \times 1.1500 \times 1.1300 \times 1.1200 \\
5.7235 =& 1.5700 \times 1.3700 \times 1.2700 \times 1.2200 \times 1.1800 \\
& \times 1.1500 \times 1.1300 \times 1.1200 \\
3.6455 =& 1.3700 \times 1.2700 \times 1.2200 \times 1.1800 \times 1.1500 \\
& \times 1.1300 \times 1.1200 \\
2.6610 =& 1.2700 \times 1.2200 \times 1.1800 \times 1.1500 \times 1.1300 \\
& \times 1.1200 \\
2.0953 =& 1.2200 \times 1.1800 \times 1.1500 \times 1.1300 \times 1.1200 \\
1.7174 =& 1.1800 \times 1.1500 \times 1.1300 \times 1.1200 \\
1.4554 =& 1.1500 \times 1.1300 \times 1.1200 \\
1.2656 =& 1.1300 \times 1.1200
\end{aligned}$$

We then compute development patterns using the chain ladder technique. The percentage of the ultimate claims observed till the end of the development period n should be estimated to enable us to compute the estimated reserve. Consequently, the inverse λ_n of these factors defines the percentage of the ultimate expected at each delay period. The inverse λ_n is computed as follows

$$\begin{aligned}
\frac{1}{12.4772} &= 0.0801; & \frac{2.1800}{12.4772} &= 0.1747 \\
\frac{1.5700}{5.7235} &= 0.2743; & \frac{1.3700}{3.6455} &= 0.3758 \\
\frac{1.2700}{2.6610} &= 0.4773; & \frac{1.2200}{2.0953} &= 0.5823 \\
\frac{1.1800}{1.7174} &= 0.6871; & \frac{1.1500}{1.4554} &= 0.7901 \\
\frac{1.1300}{1.2656} &= 0.8929
\end{aligned}$$

In Table 11, the percentage of ultimate claims still outstanding at the end of the development period n is determined by multiplying the estimated ultimate claims in Table 11 by the term $(1-\lambda_n)$. Consequently, the total estimated reserve due to the Bornheutter-Ferguson technique is then

$$\begin{aligned}
BF_{reserve} =& 963.90 + 2266.92 + 3942.54 + 6014.88 \\
& + 8467.74 + 11235.60 + 14368.86 \\
& + 17826.48 + 21525.66 = 86,612.58
\end{aligned} \tag{43}$$

Therefore, the reserve total of $86,612.58$ is necessary to settle claims. We assume a loss ratio of $\frac{90}{100}$ the premium written to estimate the naïve ultimate claim. A comparative analysis of Bornheutter-Ferguson and chain-ladder shows that

$$CL_{reserve} = 249,811.708 > BF_{reserve} = 86,612.58 \tag{44}$$

The relatively low reserve value of Bornheutter-Ferguson against the chain ladder is confirmed by the standard results in Schmidt and Zocher (2016). Following Saputra et al. (2021), we can infer that the actual loss ratio is undetermined until the last claim has been paid. The required technique of modifying the first estimate as claims evolve is acceptable; the Bornheutter-Ferguson technique combines the previous expectation of losses obtained through the loss ratio technique with the actual rate of emergence of losses and hence mitigates the impact that most recent years of losses, on the reserve and on-chain ladder estimates.

7. Conclusion

Claims reserving involves key actuarial tasks in the general insurance business, and actuaries need to forecast the ultimate claim values on various lines of business and over the entire insurance portfolio based on the observed claims of complete or incomplete development matrix values. The estimated reserves are meant to describe the liabilities of the insurance firms evolving from contingencies arising within the scope of insurance coverage. This paper estimated the reserve for outstanding claims based on the chain ladder and loss ratio with

Bornheutter-Ferguson computational principles by employing the data on cumulative paid claims covering 2010 to 2019. Since the underwriting firm is almost indeed the recipient of the risk, it is pertinent that it obliges itself to honor every genuine claim due to the insured and cover all its operating costs. Non-life insurance underwriters have long been keen on the actuarial valuation of liabilities over long-term lines of business. Although coverage is usually restricted to twelve months, issues connected with premium rating tend to be proportional to the claims experience, so claims reserve becomes a continuous phenomenon.

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