Agreement of h-functions obtained via different conformal maps

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Abstract: The h-function is an increasing and piece-wise defined, and lies on the interval [0,1] as it is given by the probability that a particle released from a basepoint z_0 in a region Ω first exit Ω within distance r of z_0. Our focusing question is: How to validate the h-function formula via different methods. Here, we provide answer to this question by computing h-function of a region for which it is possible to compute the h-function $h(r)$ analytically using three different methods. To compute $h(r)$, we use different types of conformal maps, harmonic function, and the prime function. These methods differ in whether, and how, they use the prime function. In the first method, we use the prime function twice, first in the radial-slit mapping from the unit disc D_ζ to Ω, and then in the Cayley-type map $R(\zeta)$ from D_ζ to the halfplane that is used in the construction of the harmonic function $\mathrm{Im}W(\zeta)$. In the second method, we do not use the prime function at all. Instead, the conformal map from the unit disc to Ω is given by the composition of a Joukowski map and a Möbius transformation, and the h-function is then calculated by reading off the appropriate angle of sight in the unit disc. In the third method, we use the prime function once. In this method, the conformal map from the disc to Ω is simply a Joukowski map. Meanwhile, the prime function is used in $R(\zeta)$ and hence in $\mathrm{Im}W(\zeta)$. All three methods provide the same h-function graph.

Keywords: h-functions, Harmonic measure, Conformal maps, Prime function.

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