



# PARITY CONSTRAINTS IN GRAPH LABELINGS: INVESTIGATING THE INCOMPATIBILITY OF ODD HARMONIES IN GRAPHS WITH ODD LOOPS

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**Abstract:** This paper introduces a fundamental theorem concerning the existence of odd harmonies labeling in graphs. The theorem investigates the properties of graphs with odd loops and provides insight into the relationship between graph structures and number theory. To prove the theorem, we consider graphs with at least one loop of odd order and analyze their labeling according to the concept of odd harmonies. Odd harmonies labeling involves assigning even integer labels to the vertices and satisfying specific properties related to these labels. Through rigorous mathematical reasoning and advanced graph theory concepts, we show that the existence of an odd loop within a graph leads to the impossibility of obtaining an odd harmonies labeling. The proof highlights the critical role of parity in determining the sum of edge labels and reveals the intricacies of constructing a valid labeling for such graphs. Various graph families, including cycles, bipartite graphs with odd cycles, Kneser graphs, and others, serve as illustrative examples for the theorem. The results not only unveil the limitations of odd harmonies labeling in these graphs but also shed light on the elegant interplay between graph structures and number theory. The presented theorem contributes to the deeper understanding of graph labeling and their significance in various applications.

**Keywords:** Graph families, Graph theory, Number theory, Odd harmonies labeling, Odd loops, Parity